

# PHYS 100C, LECTURE 2

Thursday, April 02, 2009  
6:30 AM

General Expression EM waves

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \hat{n}$$

$$\begin{aligned} \vec{B}(\vec{r}, t) &= \frac{1}{c} \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} (\hat{k} \times \hat{n}) = \\ &= \frac{1}{c} \hat{k} \times \vec{E} \\ \hat{n} \cdot \vec{k} &= 0 \quad (\vec{E} \perp \vec{k}, \text{transverse}) \end{aligned}$$

## Energy transport

\* Energy density  $(= \frac{U}{V})$

$$U = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right)$$

$$B = \frac{1}{c} E = \sqrt{\mu_0 \epsilon_0} E$$

$$U = \frac{1}{2} \left( \underbrace{\epsilon_0 E^2}_{\substack{\text{E-part} \\ \text{EQUAL!}}} + \underbrace{\frac{\mu_0 \epsilon_0}{\mu_0} E^2}_{\substack{\text{B-part}}} \right) = \epsilon_0 E^2$$

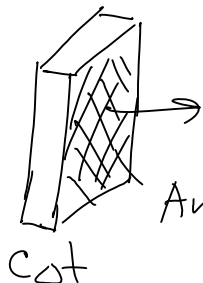
$$U = \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)$$

\* Energy transferred:

Energy flux density  
(energy per unit area, per unit time  $\frac{U}{A \cdot \Delta t}$ )  
Poynting vector:

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) = \frac{1}{\mu_0 c} E_0^2 \cos^2(kz + \omega t + \delta) \hat{z} = c u \hat{z}$$

Does this make sense?



EM wave

Area A

$c \Delta t$

Energy stored  
in volume  $A \cdot c \Delta t$   
is  $u \cdot A \cdot c \Delta t$   
will be transferred  
across A in time  $\Delta t$

$$|S(\text{flux density})| = \frac{\text{energy}}{\text{area} \cdot \text{time}} = \frac{u A \cdot c \Delta t}{A \cdot \Delta t} = u c$$

\* Momentum density (momentum/volume)

$$\vec{P} = \frac{\vec{S}}{c^2} = \frac{u}{c} \hat{z}$$

Also makes sense: Photon Energy  $\hbar \omega$   
Photon Momentum  $\frac{\hbar \omega}{c}$

We usually are interested in  
**average** values of  $S, P, u$ , etc.

Red light  $\lambda \approx 600 \text{ nm} = 6 \cdot 10^{-7} \text{ m}$

$$\text{Period } T = \frac{\lambda}{c} = \frac{6 \cdot 10^{-7} \text{ m}}{3 \cdot 10^8 \text{ m/s}} \sim 2 \cdot 10^{-15} \text{ s}$$

or 2 femtoseconds

Most experiments average over  
many  $T$ 's.

... ..

$$\langle \cos^2 \rangle = \langle \sin^2 \rangle = 1/2$$

(since  $\cos^2 + \sin^2 = 1$ )

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E^2 \quad \text{energy density}$$

$$\langle \bar{S} \rangle = \frac{1}{2} c \epsilon_0 E_0^2 \hat{z} \quad \text{energy flux}$$

$$\langle \bar{P} \rangle = \frac{\langle u \rangle}{c} \hat{z} \quad \text{momentum density}$$

Experiments usually measure  
average power per area  $\equiv$  intensity

$$I = |\langle \bar{S} \rangle| = \frac{1}{2} c \epsilon_0 E_0^2$$

Radiation pressure  $\equiv \frac{\langle \text{force} \rangle}{\text{area}}$

Pressure  $\downarrow$  momentum

$$\vec{P} = \frac{1}{A} \left| \frac{\Delta \vec{p}}{\Delta t} \right| = \frac{1}{A} \cdot \frac{\langle \bar{P} \rangle \cdot A \Delta t}{\Delta t} = \langle \bar{P} \rangle \cdot c =$$

$$= \frac{1}{2} \epsilon_0 E^2 = \frac{I}{c}$$

assuming 100% absorption.

For 100% reflection, double it

$$P = \epsilon_0 E^2 = \frac{2I}{c}$$

## \* EM waves in media:

MAXWELL EQS in media

No FREE CHARGE  $\rho = 0$

No FREE CURRENTS  $j = 0$

— —

$$\vec{\nabla} \cdot \vec{D} = 0 \quad (i)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (ii)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (iii)$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad (iv)$$

Linear media  $\vec{D} = \epsilon \vec{E}$   
 $\vec{H} = \frac{1}{\mu} \vec{B}$

Homogeneous  $\mu, \epsilon$  indep. of  $x, y, z$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (i)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (ii)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (iii)$$

$$\vec{\nabla} \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t} \quad (iv)$$

Same as MAXWELL EQ'S FOR VACUUM

BUT  $\mu_0 \rightarrow \mu$   
 $\epsilon_0 \rightarrow \epsilon$

Solutions: EM waves

with velocity  $v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{n}$

where  $n = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}}$  index of refraction

$\mu \approx \mu_0$  for most materials

$n \approx \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\epsilon_R} \leftarrow \text{dielectric constant}$

$\epsilon_R = \frac{\epsilon}{\epsilon_0} > 1$  (almost always)

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} > 1 \quad (\text{almost always})$$

$$n > 1 \quad \text{and} \quad v < c$$

Similarly, energy density:

$$u = \frac{1}{2} \left( \epsilon E^2 + \frac{1}{\mu} B^2 \right)$$

$$\vec{B} = \frac{1}{v} \hat{k} \times \vec{E} \quad (\text{transverse field})$$

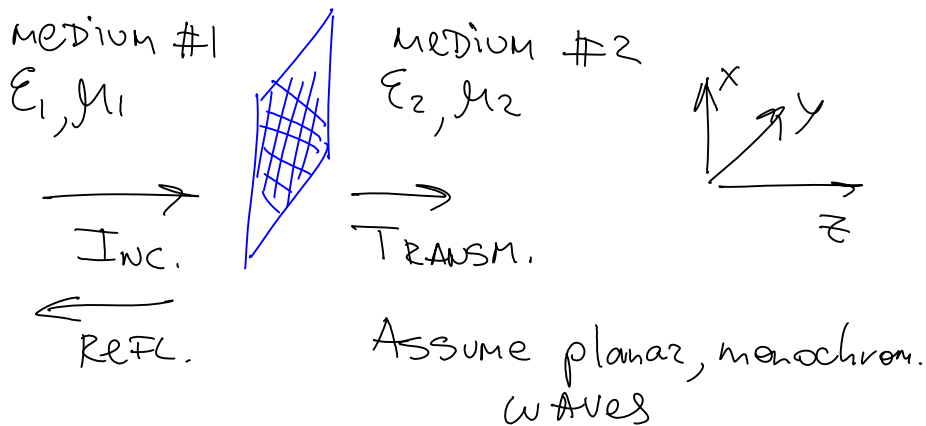
$\Downarrow$

$$u = \epsilon E^2$$

$$\vec{S} = \frac{1}{\mu} (\vec{E} \times \vec{B}) = v \cdot u \cdot \hat{k}$$

$$I = \langle \vec{S} \rangle = \frac{1}{2} \epsilon v E_0^2$$

## \* Reflection/Transmission at Normal Incidence:



\* BOUNDARY CONDITIONS:

From Maxwell Eq. (i)-(iv):

$$D_1^\perp = D_2^\perp \quad \text{or} \quad \epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp \quad (i)$$

$$B_1^\perp = B_2^\perp \quad (ii)$$

$$\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel} \quad (\text{iii})$$

$$\vec{H}_1^{\parallel} = \vec{H}_2^{\parallel} \quad \text{or} \quad \frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2} \quad (\text{iv})$$

\* FOR NORMAL INCIDENCE,

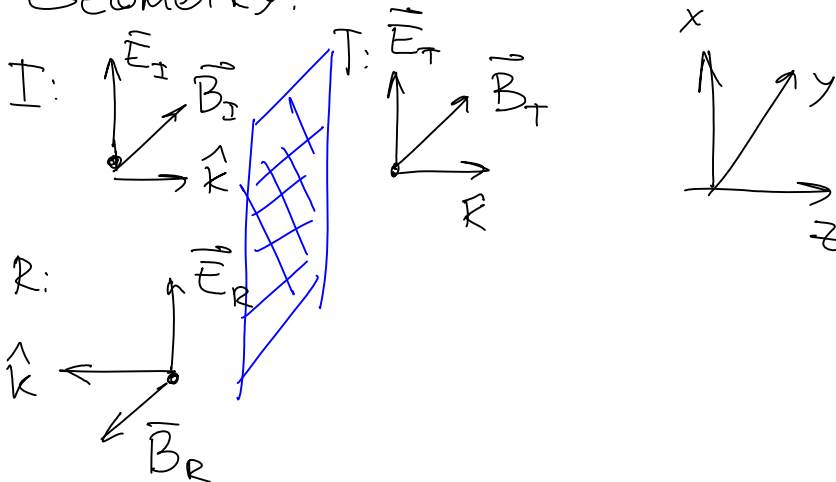
INCIDENT (I) WAVE:

$$\vec{E}_I(z, t) = E_{0I} e^{i(kz - \omega t)} \hat{x}$$

$$\vec{B}_I(z, t) = \frac{E_{0I}}{v_1} e^{i(kz - \omega t)} \hat{y}$$

$$\text{Where } v_1 = \frac{1}{\sqrt{\mu_1 \epsilon_1}}$$

\* GEOMETRY:



\* REFLECTED WAVE: (R)

$$\vec{E}_R(z, t) = E_{0R} e^{i(-kz - \omega t)} \hat{x}$$

$$\vec{B}_R(z, t) = -\frac{E_{0R}}{v_1} e^{i(-kz - \omega t)} \hat{y}$$

NOTE "-" sign for  $k_1$  in exp.

as well as for  $\vec{B}_R$  direction.

\* Transmitted wave: (T)

$$\vec{E}_T(z, t) = E_{0T} e^{i(k_z z - \omega t)} \hat{x}$$

$$\vec{B}_T(z, t) = \frac{E_{0T}}{v_2} e^{i(k_z z - \omega t)} \hat{y}$$

$$\omega = k_1 v_1 = k_2 v_2 = \text{const. across boundary}$$

In the class we stopped here.  
We will continue derivations below  
ON APRIL 7th

BOUNDARY CONDITIONS  
FOR  $z=0$

$$E_{0I} + E_{0R} = E_{0T} \quad (\text{iii})$$

$$\frac{1}{\mu_1} \frac{1}{v_1} E_{0I} - \frac{1}{v_1} E_{0R} = \frac{1}{\mu_2} \left( \frac{1}{v_2} E_{0T} \right) \quad (\text{iv})$$

(i) AND (ii) are NOT USEFUL  
SINCE  $E^\perp = B^\perp = 0$  (NORMAL INCIDENCE)

(iv) REWRITTEN:

$$E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} \quad (\text{iv})$$

$$\mu_2 v_2$$

\* Note: IF we multiply (iii) by (iv)

$$E_{OI}^2 - E_{OR}^2 = \frac{\mu_1 v_1}{\mu_2 v_2} E_{OT}^2, \text{ OR:}$$

$$\frac{1}{\mu_1 v_1} E_{OI}^2 = \frac{1}{\mu_1 v_1} E_{OR}^2 + \frac{1}{\mu_2 v_2} E_{OT}^2$$

$$\frac{1}{2} \epsilon_1 v_1 E_{OI}^2 = \frac{1}{2} \epsilon_1 v_1 E_{OR}^2 + \frac{1}{2} \epsilon_2 v_2 E_{OT}^2$$

$\uparrow$  Incident Intensity       $\uparrow$  REFLECTED       $\uparrow$  TRANSM.

ENERGY IS CONSERVED

\* IF we ADD (i) + (ii);

and use

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

$$E_{OT} = \frac{2}{1+\beta} E_{OI}$$

$$E_{OR} = \frac{1-\beta}{1+\beta} E_{OI}$$

If:  $\beta < 1$   $E_{OR} > 0$ ,

REFL. is IN PHASE w/ Incident

$\beta > 1$   $E_{OR} < 0$

REFL. wave is  $180^\circ$  out of phase

Since  $\mu \sim \mu_0$   $\beta \approx \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$E_{OT} \approx \left| \frac{2n_1}{n_1 + n_2} \right| E_{OI}$$



$$0.1 \quad |n_1 + n_2| \quad -1$$

$$E_{OR} \cong \left| \frac{n_1 - n_2}{n_1 + n_2} \right| E_{OI}$$

Reflection coefficient:

$$R \equiv \frac{I_R}{I_I} = \left( \frac{E_{OR}}{E_{OI}} \right)^2 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$T \equiv \frac{I_T}{I_I} = \left( \frac{E_{OT}}{E_{OI}} \right)^2 \cdot \frac{\epsilon_2 V_2}{\epsilon_1 V_1} = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

\* Note THAT  $R + T = 1$